

Power-law entropy corrected holographic and new agegraphic $f(R)$ -gravity models

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Abstract

Motivated by the recent works of us [1], we establish a correspondence between $f(R)$ -gravity with the power-law entropy corrected holographic dark energy (PLECHDE) and new agegraphic dark energy (PLECNADE) models. We reconstruct corresponding $f(R)$ -gravities and obtain the equation of state parameters. We show that the selected $f(R)$ -gravity models can accommodate the transition from the quintessence state to the phantom regime as indicated by the recent observations.

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1 The theory of $f(R)$ modified gravity

The Friedmann equations in $f(R)$ -gravity are given by [2]

$$\frac{3}{k^2}H^2 = \rho_m + \rho_R, \quad (1)$$

$$\frac{1}{k^2}(2\dot{H} + 3H^2) = -(p_m + p_R), \quad (2)$$

where

$$\rho_R = \frac{1}{k^2} \left[-\frac{1}{2}f(R) + 3(\dot{H} + H^2)f'(R) - 18(4H^2\dot{H} + H\ddot{H})f''(R) \right], \quad (3)$$

$$p_R = \frac{1}{k^2} \left[\frac{1}{2}f(R) - (\dot{H} + 3H^2)f'(R) + 6(8H^2\dot{H} + 6H\ddot{H} + 4\dot{H}^2 + \ddot{H})f''(R) + 36(H\ddot{H} + 4H^2\dot{H})^2f'''(R) \right], \quad (4)$$

with

$$R = 6(\dot{H} + 2H^2). \quad (5)$$

Here, ρ_m and p_m are the energy density and pressure of the matter, respectively. Also ρ_R and p_R are due to the contribution of $f(R)$ -gravity.

The equation of state (EoS) parameter of the curvature contribution is defined as [3]

$$\omega_R = \frac{p_R}{\rho_R} = -1 + \frac{p_R + \rho_R}{\rho_R}. \quad (6)$$

For a given $a = a(t)$, by the help of Eqs. (3) and (4) one can reconstruct the $f(R)$ -gravity according to any DE model given by the EoS $p_R = p_R(\rho_R)$ or $\rho_R = \rho_R(a)$ [4].

Here we consider the two classes of scale factors, the first one is given by [4, 5]

$$a(t) = a_0(t_s - t)^{-h}, \quad t \leq t_s, \quad h > 0. \quad (7)$$

Using Eqs. (5) and (7) one can obtain

$$H = \frac{h}{t_s - t} = \left[\frac{h}{6(2h + 1)} R \right]^{1/2}, \quad \dot{H} = H^2/h. \quad (8)$$

For the second class of scale factors defined as [4]

$$a(t) = a_0 t^h, \quad h > 0, \quad (9)$$

one can get

$$H = \frac{h}{t} = \left[\frac{h}{6(2h - 1)} R \right]^{1/2}, \quad \dot{H} = -H^2/h. \quad (10)$$

In sections 2 to 4 using the two classes of scale factors (7) and (9), we reconstruct different $f(R)$ -gravities according to the PLECHDE and PLECNADÉ models.

2 PLECH $f(R)$ -gravity model

The holographic dark energy (HDE) density is given by [6]

$$\rho_\Lambda = \frac{3M_p^2 c^2}{L^2}, \quad (11)$$

where c and $M_p^{-2} = 8\pi G$ are the numerical constant and reduced Planck mass. Also L is the IR-cutoff of the universe. This model can be modified due to the power-law corrections to the entropy which appears in dealing with the entanglement of quantum fields in and out the horizon [7]. The power-law corrected entropy takes the form [7]

$$S = \frac{A}{4G} [1 - K_\alpha A^{1-\alpha/2}], \quad (12)$$

where α is a dimensionless constant whose value is currently under debate and

$$K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}}. \quad (13)$$

Also r_c is the crossover scale. Using (12), the PLECHDE density was introduced as [8]

$$\rho_\Lambda = M_p^2 \left(\frac{3c^2}{L^2} - \frac{\beta}{L^\alpha} \right), \quad (14)$$

where in the absence of correction terms ($\alpha = \beta = 0$) yields the well-known HDE density [7].

For a flat universe we have $L = R_h$, where R_h is the event horizon defined as

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}. \quad (15)$$

For the first class of scale factors (7) and using Eq. (8), the future event horizon R_h yields

$$R_h = a \int_t^{t_s} \frac{dt}{a} = \frac{t_s - t}{h+1} = \frac{1}{h+1} \sqrt{\frac{6h(2h+1)}{R}}. \quad (16)$$

Here due to having $R_h > 0$ then $-1 < h$.

Inserting Eq. (16) into (14) one can get

$$\rho_\Lambda = M_p^2 \left[\frac{c^2(h+1)^2}{2h(2h+1)} R - \frac{\beta(h+1)^\alpha}{(6h(2h+1))^{\frac{\alpha}{2}}} R^{\frac{\alpha}{2}} \right]. \quad (17)$$

Equating (3) with (17) gives

$$\begin{aligned} & 2R^2 f''(R) - (h+1)Rf'(R) + (2h+1)f(R) \\ & + \left[\frac{c^2(h+1)^2}{h} R - \frac{2\beta(2h+1)(h+1)^\alpha}{(6h(2h+1))^{\frac{\alpha}{2}}} R^{\frac{\alpha}{2}} \right] = 0. \end{aligned} \quad (18)$$

Solving Eq. (18) yields the PLECHDE $f(R)$ -gravity as

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{c^2(h+1)^2 R}{h^2} - \frac{4\beta(2h+1)(h+1)^\alpha R^{\frac{\alpha}{2}}}{(h(\alpha-4) - (\alpha-1)(\alpha-2))(6h(2h+1))^{\frac{\alpha}{2}}}, \quad (19)$$

where

$$m_{\pm} = \frac{3+h \pm \sqrt{h^2 - 10h + 1}}{4}. \quad (20)$$

Also λ_{\pm} are the integration constants which can be found from the following boundary conditions [9]

$$f(R_0) = -2R_0, \quad (21)$$

$$f'(R_0) \sim 0, \quad (22)$$

where $R_0 \sim (10^{-33}\text{eV})^2$ is the current curvature.

Applying the above boundary conditions to the solution (19) one can obtain λ_{\pm} as

$$\lambda_{\pm} = \frac{1}{(m_{\pm} - m_{\mp})R_0^{m_{\pm}}} \left[\left(2m_{\mp} + \frac{(1 - m_{\mp})c^2(h+1)^2}{h} \right) R_0 + \frac{2\beta(\alpha - 2m_{\mp})(2h+1)(h+1)^\alpha R_0^{\frac{\alpha}{2}}}{(h(\alpha-4) - (\alpha-1)(\alpha-2))(6h(2h+1))^{\frac{\alpha}{2}}} \right]. \quad (23)$$

Replacing Eq. (19) into (6) and using (8) and (16) one can get the EoS parameter of the PLECHDE $f(R)$ -gravity model as

$$\omega_R = -1 - \frac{2}{3h} \left[\frac{\alpha X - 1}{2X - 1} \right], \quad (24)$$

where

$$X = \frac{\beta(h+1)^{\alpha-2}}{6c^2} \left(\frac{R}{6h(2h+1)} \right)^{\frac{\alpha-2}{2}} = \frac{\beta(h+1)^{\alpha-2}}{6c^2} \left(\frac{1}{1+z} \right)^{\frac{\alpha-2}{h}}, \quad (25)$$

and $z = \frac{1}{a} - 1$ is the redshift. We take $a_0 = 1$ for the present value of the scale factor.

For $\alpha < 0$, $\beta < 0$ and $-1 < h$, ($h \neq 0$) one can find that the EoS parameter (24) can cross the phantom-divide line.

For the second class of scale factors, the resulting $f(R)$ is

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{c^2(1-h)^2 R}{h^2} + \frac{4\beta(2h-1)(1-h)^\alpha R^{\frac{\alpha}{2}}}{(h(\alpha-4) + (\alpha-1)(\alpha-2))(6h(2h-1))^{\frac{\alpha}{2}}}, \quad (26)$$

where

$$m_{\pm} = \frac{3-h \pm \sqrt{h^2 + 10h + 1}}{4}. \quad (27)$$

3 PLECNA $f(R)$ -gravity model

The new agegraphic dark energy (NADE) density is given by [10]

$$\rho_\Lambda = \frac{3n^2 M_p^2}{\eta^2}, \quad (28)$$

where η is the conformal time defined as

$$\eta = \int_0^a \frac{dt}{Ha^2}. \quad (29)$$

The power-law entropy corrected version of the NADE density takes the form [8]

$$\rho_\Lambda = M_p^2 \left(\frac{3n^2}{\eta^2} - \frac{\beta}{\eta^\alpha} \right). \quad (30)$$

For the first class of scale factors (7) and using Eq. (8), the conformal time η yields

$$\eta = \int_t^{t_s} \frac{dt}{a} = \frac{(t_s - t)^{h+1}}{a_0(h+1)} = \frac{1}{a_0(h+1)} \left(\frac{6h(2h+1)}{R} \right)^{\frac{h+1}{2}}. \quad (31)$$

Note that due to having $\eta > 0$ then $-1 < h$.

Substituting Eq. (31) into (30) one can get

$$\rho_\Lambda = M_p^2 \left[\left(\frac{3a_0^2 n^2 (h+1)^2}{(6h(2h+1))^{h+1}} \right) R^{h+1} - \left(\frac{\beta a_0^2 (h+1)^{2\alpha}}{(6h(2h+1))^{\frac{\alpha(h+1)}{2}}} \right) R^{\frac{\alpha}{2}} \right]. \quad (32)$$

Equating (3) with (32) gives

$$2R^2 f''(R) - (h+1)Rf'(R) + (2h+1)f(R) + \left(\frac{a_0^2 n^2 (h+1)^2}{h(6h(2h+1))^h} \right) R^{h+1} - \left(\frac{2\beta a_0^2 (2h+1)(h+1)^{2\alpha}}{(6h(2h+1))^{\frac{\alpha(h+1)}{2}}} \right) R^{\frac{\alpha(h+1)}{2}} = 0. \quad (33)$$

Solving Eq. (33) yields the PLECADE $f(R)$ -gravity as

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} - \frac{a_0^2 n^2 (h+1)^2 R^{h+1}}{h^2 (h+2) (6h(2h+1))^h} + \frac{4\beta a_0^2 (2h+1)(h+1)^\alpha R^{\frac{\alpha(h+1)}{2}}}{(6h(2h+1))^{\frac{\alpha(h+1)}{2}} ((2-3\alpha+\alpha^2) + 2h(2-2\alpha+\alpha^2) + \alpha(\alpha-1)h^2)}, \quad (34)$$

where m_\pm are given by (20). Also λ_\pm are determined from the boundary conditions (21) and (22) as

$$\lambda_\pm = \frac{1}{(m_\pm - m_\mp) R_0^{m_\pm}} \left[2m_\mp R_0 + \frac{a_0^2 n^2 (h+1)^2 (h+1 - m_\mp) R_0^{h+1}}{h^2 (h+2) (6h(2h+1))^h} - \frac{2\beta a_0^2 (2h+1)(h+1)^{2\alpha} (\alpha(h+1) - 2m_\mp) R_0^{\frac{\alpha(h+1)}{2}}}{(2(2h+1) - (h+1)(h+3)\alpha + (h+1)^2 \alpha^2) (6h(2h+1))^{\frac{\alpha(h+1)}{2}}} \right]. \quad (35)$$

Replacing Eq. (34) into (6) and using (8) and (31) one can get the EoS parameter of the PLECNAD E $f(R)$ -gravity model as

$$\omega_R = -1 - \frac{2(h+1)}{3h} \left[\frac{\alpha X - 1}{2X - 1} \right], \quad (36)$$

where

$$X = \frac{a_0^{(\alpha-2)} \beta (h+1)^{(\alpha-2)}}{6n^2} \left(\frac{R}{6h(2h+1)} \right)^{\frac{(\alpha-2)(h+1)}{2}} = \frac{\beta (h+1)^{(\alpha-2)}}{6n^2} \left(\frac{1}{1+z} \right)^{\frac{(\alpha-2)(h+1)}{h}}. \quad (37)$$

Equation (36) shows that for $\alpha < 0$, $\beta < 0$ and $-1 < h$, ($h \neq 0$) crossing the phantom-divide line can occur.

For the second class of scale factors we find

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \frac{a_0^2 n^2 (h-1)^2 (6h(2h-1))^h R^{1-h}}{h^2 (h-2)} - \frac{4\beta a_0^2 (2h-1)(h-1)^{2\alpha} R^{\frac{\alpha(1-h)}{2}}}{(6h(2h-1))^{\frac{\alpha(1-h)}{2}} ((2-3\alpha+\alpha^2) - 2h(2-2\alpha+\alpha^2) + \alpha(\alpha-1)h^2)}, \quad (38)$$

where m_{\pm} are given by Eq. (27).

4 $f(R)$ reconstruction in de Sitter space

The scale factor in de sitter space is defined as

$$a(t) = a_0 e^{Ht}, \quad H = \text{constant}, \quad (39)$$

which can describe the early-time inflation of the universe [4]. Using Eqs. (5) and (39) one can obtain

$$H = \left(\frac{R}{12} \right)^{1/2}. \quad (40)$$

Then Eqs. (3) and (4) take the forms

$$\begin{aligned} k^2 \rho_R &= -\frac{1}{2} f(R) + 3H^2 f'(R), \\ k^2 p_R &= \frac{1}{2} f(R) - 3H^2 f'(R). \end{aligned} \quad (41)$$

Also the EoS parameter yields $\omega_R = \frac{p_R}{\rho_R} = -1$ which behaves like the cosmological constant.

4.1 PLECHDE model

For the scale factor (39), using Eq. (40) the future event horizon R_h reduces to

$$R_h = a \int_t^\infty \frac{dt}{a} = H^{-1} = \left(\frac{R}{12} \right)^{-1/2}. \quad (42)$$

Substituting Eq. (42) into PLECHDE density (14) yields

$$Rf'(R) - 2f(R) - c^2R + 4\beta\left(\frac{R}{12}\right)^{\frac{\alpha}{2}} = 0. \quad (43)$$

This gives

$$f(R) = \lambda R^2 - c^2R - \frac{8\beta}{\alpha - 4}\left(\frac{R}{12}\right)^{\frac{\alpha}{2}}. \quad (44)$$

where λ is an integration constant. Also from $\omega_R = \frac{p_R}{\rho_R} = -1$ and continuity equation for PLECHDE one can get

$$\beta = \frac{c^2R}{2\alpha}\left(\frac{12}{R}\right)^{\frac{\alpha}{2}}. \quad (45)$$

Finally one can rewrite (44) as

$$f(R) = \lambda R^2 - \frac{c^2(\alpha - 2)^2}{\alpha(\alpha - 4)}R. \quad (46)$$

Note that the term R^2 confirms that the model (46) satisfies the inflation condition [11].

4.2 PLECADE model

For PLECADE, the conformal time η for the scale factor (39) yields

$$\eta = \int_0^t \frac{dt}{a} = \frac{1}{a_0 H} (1 - e^{-Ht}). \quad (47)$$

Here to obtain $\eta = \eta(R)$ one cannot replace t by R in (47). Therefore for the scale factor (39) one cannot obtain the $f(R)$ -gravity models corresponding to the PLECADE density (30). To avoid of this problem we set $t \rightarrow \infty$ for the upper limit of the integral (47). Hence the result yields

$$\eta = \int_0^\infty \frac{dt}{a} = (a_0 H)^{-1} = \left(\frac{a_0^2 R}{12}\right)^{-1/2}. \quad (48)$$

Substituting Eq. (48) into PLECADE density (30) and using Eq. (41) one can obtain

$$Rf'(R) - 2f(R) - a_0^2 n^2 R + 4\beta\left(\frac{a_0^2 R}{12}\right)^{\frac{\alpha}{2}} = 0. \quad (49)$$

Solving the above differential equation yields

$$f(R) = \lambda R^2 - a_0^2 n^2 R - \frac{8\beta}{\alpha - 4}\left(\frac{a_0^2 R}{12}\right)^{\frac{\alpha}{2}}, \quad (50)$$

where λ is an integration constant. Also from $\omega_R = \frac{p_R}{\rho_R} = -1$ and continuity equation for PLECADE one can get

$$\beta = \frac{a_0^2 n^2 R}{2\alpha}\left(\frac{12}{a_0^2 R}\right)^{\frac{\alpha}{2}}, \quad (51)$$

and we finally find

$$f(R) = \lambda R^2 - \frac{a_0^2 n^2 (\alpha - 2)^2}{\alpha(\alpha - 4)}R. \quad (52)$$

Here like the model (46) the inflation condition is respected because of the term R^2 .

5 Conclusions

Here, we investigated the power-law entropy corrected versions of the HDE and NADE in the framework of $f(R)$ modified gravity. We reconstructed different $f(R)$ -gravity models corresponding to the PLECHDE and PLECNAD models. We obtained the EoS parameters of the PLECH and PLECNAD $f(R)$ -gravity models. Our calculations show that for the selected models, the transition from the quintessence state to the phantom regime can occur which is compatible with the recent observations. Furthermore, we studied the PLECH and PLECNAD $f(R)$ -gravity models in de Sitter space. We concluded that these models can also satisfy the inflation condition.

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